1. Introduction

The “approach flow” is the section of a paper machine between the machine chest and the headbox. It is made up of a number of unit operations shown in Figure 1. The functions of these various operations are described below.

**Machine Chest**

This is the last storage tank before the paper machine. It serves, as all the other tanks do, to attenuate low frequency variations in stock. However, its main function is to hold a reserve of stock for the paper machine (about 20 minutes operating time) in case upstream operations must be closed down. The “thick stock” is at 3%, the typical consistency for storage and pumping before a paper machine.

**Consistency Control**

There is usually an on-line consistency measurement and control using dilution by whitewater after the machine chest.
**Tickler Refining**

This is a light refining to eliminate fibre bundles and flocs, and sometimes to lower the freeness. It is usually accomplished in a conical refiner having a coarse bar pattern, called a “Jordan”.

**Stuff Box**

This provides a constant back pressure for the fan pumps to maintain stable flow to the paper machine. It is in essence a constant head tank operating on the overflow principle illustrated in Figure 2.

![Figure 2](image)

**Basis Weight Meter and Valve**

This meter and valve control the amount of fibre fed to the paper machine. This in turn controls the amount of paper coming off the reel at the end of the paper machine. It is the most important valve on a paper machine.
**Fan Pump**

This large centrifugal pump delivers stock to the paper machine. It is located just after the point of dilution of the thick stock with white water. Indeed, the fan pump is often used to mix these two streams. After dilution with white water, the stock flowing towards the headbox is at a consistency of 0.5-1%, ready for papermaking.

**Cleaning and Screening**

After the fan pump, stock is passed through one or more stages of cleaners (hydrocyclones) and then through a pressure screen before reaching the headbox. The aim here is to remove debris that may damage the paper machine and lower paper quality. Because cleaners impose a large head loss, a “secondary” fan pump is sometimes necessary between the cleaners and screen to restore the pressure.

**Deculators**

These devices remove air from the stock. Such air is detrimental to paper quality and can affect operation, e.g. cause pump cavitation. Deculators are in essence chambers under vacuum, typically located in the accept line coming from the cleaners as shown in Figure 3.
The stock arrives at the paper machine in a pipe and must be distributed uniformly over the width of the machine. This is a formidable task. Paper machines can be up to 10 m wide, the equivalent of 50,000 fibre lengths, but paper is only about 5 fibres thick.

Modern headboxes use a tapered manifold design for the flow distributor illustrated in Figure 4. Here stock enters on one side of the paper machine and is drawn off over the PM width into a tube bank. At the downstream side of the PM there is an overflow valve, which the operator can adjust to maintain constant pressure along the distributor. This is monitored by the absence of flow in a sight glass, shown in Figure 5. The effect of overflow is illustrated in Figure 6. Typically, the overflow is about 10% of the entry flow.

The shape of the manifold is critical in achieving a uniform pressure across the PM. In the absence of friction, this shape is a linearly decreasing area, as shown in Figure 7a. However, pressure losses caused by friction always exist. One source is wall friction. To
account for this, the shape of the manifold must assume a complex exponential shape, as shown in Figure 7. Another pressure loss occurs from boundary layer separation at the entries to the tube bank, giving a “loss coefficient”. This requires the manifold shape to be even more complex to maintain constant pressure.

Modern manifolds for paper machines are designed by equipment manufacturers. Machine operators have only the overflow valve to adjust to maintain constant pressure. Opening or closing the overflow valve for this purpose is governed by the Bernoulli equation.

In addition to a uniform pressure, which gives uniform velocity entering the headbox, the distributor must also straighten the flow, to align it in the machine direction. This is accomplished in the tube bank through which stock flows into the paper machine. The tubes must be of some length, typically about 7 tube diameters, as shown in Figure 5.
Figure 5

Figure 6
In order to have the same flow leaving the distributor at all positions, \( x \), you need to ensure that the pressure drop across the width of the headbox is constant and since the headbox exits to the atmosphere that means that the pressure, \( P(x) \), must be constant.

Continuity:

\[
\frac{d}{dx}(A(x)V(x)) = -V_H \, dx
\]

From the energy equation assuming no friction loss across distributor

\[
P(x) + \rho V(x)^2 = \text{constant}
\]

From the discussion above we know that \( V(x) \) is a constant then from energy we see that \( V(x) \) must also be a constant

\[P(x) = \text{constant}, \quad V(x) = \text{constant} = V_1\]
\[ V_1 dA(x) = -V_H \, dx \]

\[ A(x) = A_1 - \frac{V_H}{V_1} \, x \]

We see that if there is no friction in the distributor then we get a linear distributor shape.

However, we know that there is friction across the distributor because if there wasn’t we would have flow with no pressure drop. If now consider frictional flow in the distributor:

In general the head loss due to friction is given by the following.

\[ h_c = f \frac{L}{D} \rho V^2 \]

We should note that is assumes that the friction factor is constant which, as we know from the moody curve for pipe flow, is only true for large reynold number, fully turbulent flow.

Incremental form

\[ dh_L = \frac{f}{D_m} \rho V^2_{(x)} \, dx \]

Inserting into the energy equation
\[
d\left( P(x) + \rho V^2(x) \right) = -\frac{f}{D_m} \rho V^2(x) \, dx
\]

Again imposing that \( P(x) \) is a constant:

\[
d \rho \frac{V^2(x)}{V^2(x)} = -f \frac{V(x)}{D_m} \, dx
\]

\[
\frac{d V^2(x)}{V^2(x)} = -\frac{f}{D_m} \, dx
\]

Integrating …

\[
\ln V^2(x) = -\frac{fx}{D_m} + C_1
\]

Rearranging:

\[
V^2(x) = C_1 e^{-\frac{\rho x}{D_m}}
\]

Applying the boundary condition:

\[
x = 0 \quad V(0) = V_1 \quad \therefore C_1 = V_1^2
\]

Yields:
\[
V(x) = V_1 e^{2D_m} \frac{-fx}{m}
\]

Now we insert this back into the continuity equation to get \( A(x) \) and solve as we did before.

\[
d\left( A(x)V_1 e^{2D_m} \right) = -V_H dx
\]

\[
A(x) V_1 e^{2D_m} = -V_H x + C_1
\]

at \( x = 0 \) \( A(0) = A_1 \)

\[
A_1 V_1 = C_1
\]

\[
A(x) = \left( A_1 \frac{-V_H}{V_1} x \right) \frac{fx}{\ell^{2D_m}}
\]
3.0 White Water System Material Balance

To better understand the papermachine wet end and its response to changes in operating conditions requires a knowledge of the material balance of all components of the pulp stock furnished to the machine.

A simplified view of the paper machine and its associated flows is given in the diagram below:

The amount going to the sewer has decreased over time.

Typical values of the flow for an older machine are:

- Sewer flow: 200 m³/ton
- Concentration 1.5 kg/m³
- Material lost is … 300Kg/ton of paper.

For a modern machine:

- Sewer flow: 10 m³/ton
- Concentration 0.05 kg/m³
• Material lost is ... 0.5 Kg/ton of paper.

In the above figure it is shown how material circulates through the backwater system. What is important is the pulp is diluted significantly by the short recirculation and the amount of recycle doesn’t affect the final mass production on the machine.

**Retention**

\[ C_0 V_0 = C_1 V_1 + C_f V_f \]

Then we can define the retention, R, as
\[ R = \frac{C_f V_f}{C_0 V_0} \]

Or

\[ R = 1 - \frac{C_1 V_1}{C_0 V_0} \]

Note that in this example,

\[ V_1 \approx V_0 \]

So,

\[ R = 1 \approx \frac{C_1}{C_0} \]

On the papermachine we must consider retention at two different points as water is collected separately. That is,
Short circulation

\[ Q_1 C_1 = Q_2 C_v + Q_3 C_3 \]

If we define,

\[ R_s = \frac{Q_3 C_3}{Q_1 C_1} \]

Then,

\[ 1 - R_s = \frac{Q_2 C_v}{Q_1 C_1} \]

But for a typical machine,

\[ Q_1 \approx Q_2 \]
So,

\[ 1 - R_s \approx \frac{C_v}{C_1} \]

This is called the first pass retention.

*Long Circulation*

Similarly we can derive the same thing for the long circulation.

\[ Q_3 C_3 = Q_5 C_5 + Q_4 C_4 \]

If we define,

\[ R_L = \frac{Q_4 C_4}{Q_3 C_3} \]

\[ 1 - R_L = \frac{Q_5 C_5}{Q_3 C_3} \]

*Total wire retention*

The total wire retention can be calculated as:

\[ Q_1 C_1 = Q_4 C_4 + Q_5 C_5 + Q_2 C_v \]
\[
1 = \frac{Q_4 C_4}{Q_1 C_1} + \frac{Q_5 C_5}{Q_1 C_1} + \frac{Q_2 C_v}{Q_1 C_1}
\]

We define,

\[
R_l \equiv \frac{Q_4 C_4}{Q_1 C_1}
\]

Then, we know that,

\[
1 - R_s = \frac{Q_2 C_v}{Q_1 C_1}, \quad R_s (1 - R_L) = \frac{Q_5 C_5}{Q_1 C_1},
\]

So we get,

\[
1 = R_T + R_s (1 - R_L) + (1 - R_s) + (1 - R_s)
\]

\[
R_T = R_s R_L
\]

Examples:
Dynamics of White Water Systems

So far we have only considered the steady state problem. However, we can also have unsteady situations, for example during start up or when you have grade changes that involve basis wt changes. Often a paper machine makes a series of grades of paper where each grade has a different basis weight (mass per unit area of paper). The basis weight changes can be made by increasing or decreasing the consistency entering the paper machine, usually stepped up or down through the grades.

We can analyze this problem by the same approach but assuming that the white water tank is a CSTR.

Assume that all flows are constant and there is no fibre in the tank at \( t=0 \) hence \( C_2(0)=0 \).

Feed to the headbox is

\[
= Q_0 C_0 + Q_2 C_2
\]

By definition,
\[ 1 - R_T = \frac{Q_2 C_v}{Q_0 C_0 + Q_2 C_2} \]

\[ R_s = \frac{Q_3 C_3}{Q_1 C_1} = \frac{Q_3 C_3}{Q_0 C_0 + Q_2 C_2} \]

A mass balance on the tank (CSTR) yields,

\[ V \frac{dC_2}{dt} = Q_2 C_v - Q_2 C_2(t) \]

By substitution,

\[ V \frac{dC_2}{dt} + R_T Q_2 C_2(t) = (1 - R) Q_0 C_0 \]

Remembering our Differential equations we need a homogeneous solution and a particular solution

\[ C_2(t) = C_{2H} + C_{2P} \]

\[ V \frac{dC_{2H}}{dt} + R_T Q_2 C_2(t) = 0 \]
\[ C_{2H} = A \exp\left(- \frac{R_T Q_2}{V} t \right) \]

\[ C_{2P} = \frac{1 - R_T}{R_T} \frac{Q_0}{Q_2} C_0 \]

\[ C_2(t) = A \exp\left(- \frac{R_T Q_2}{V} t \right) + \frac{1 - R_T}{R_T} \frac{Q_0}{Q_2} C_0 \]

The Boundary condition is \( C_2(0) = 0 \).

\[ A = \frac{1 - R_T}{R_T} \frac{Q_0}{Q_2} C_0 \]

\[ C_2(t) = \frac{1 - R_T}{R_T} \frac{Q_0}{Q_2} C_0 \left[ 1 - A \exp\left(- \frac{R_T Q_2}{V} t \right) \right] \]

Since all flow are constant,

\[ Q_0 = Q_3 \]

We can calculate the consistency of the pulp leaving the papermachine. After a little work we get:
\[
\frac{C_3(t)}{C_0} = 1 - (1 - R_T) \exp \left( - \frac{R_T Q_2}{V} t \right)
\]

This means that it takes a certain amount of time for the flow to come to steady state and that time is essentially the time constant in the exponential, that is,

\[
\tau = \frac{V}{R_T Q_2}
\]

Thus, the time for the flow to come to steady state is proportional to the size of the whitewater tank, and inversely proportional to the Retention and the flow rate.